Province of the
EASTERN CAPE
EDUCATION

MATHEMATICS P2

COMMON TEST

JUNE 2014

NATIONAL SENIOR CERTIFICATE

GRADE 12

Marks: 125

Time: 2½ hours

N.B. This question paper consists of 9 pages, 2 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of SIX questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. A diagram sheet for answering QUESTION 4.1, QUESTION 5.2, QUESTION 6.1, QUESTION 6.2 and QUESTION 6.3 is attached at the end of this question paper. Write your name on these diagram sheets in the spaces provided and insert the diagram sheets inside the back cover of your ANSWER BOOK.

8. Diagrams are NOT necessarily drawn to scale.

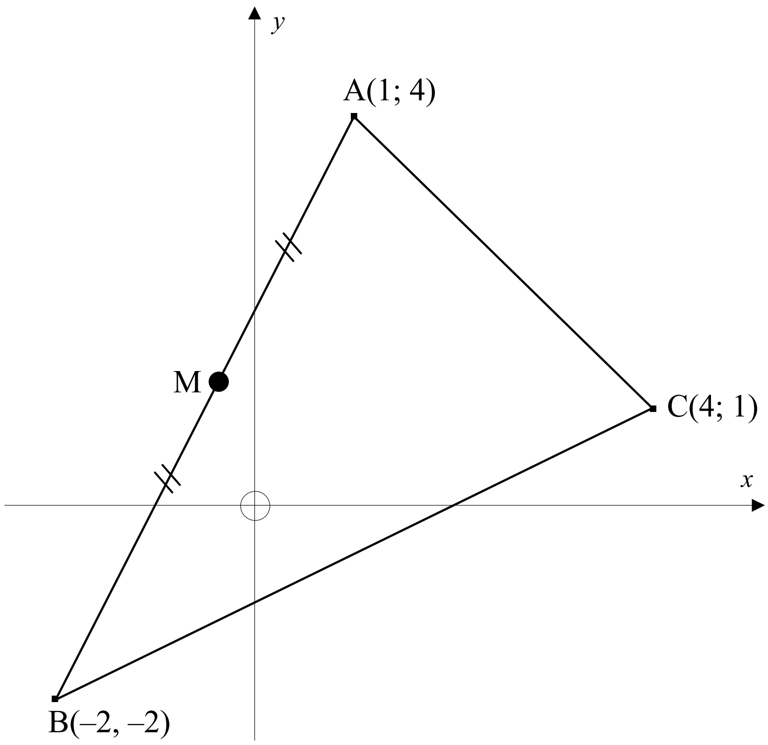
9. An information sheet with formulae is included at the end of this question paper.

10. Number the answers correctly according to the numbering system used in this question paper.

11. Write neatly and legibly.

QUESTION 1

1.1 A(1; 4), B(–2; –2) and C(4; 1) are vertices of Δ ABC in the cartesian plane as shown in the figure below. M is the midpoint of AB.



1.1.1 Calculate the length of AC (leave the answer in simplified surd form) (3)

1.1.2 Determine the co-ordinates of M. (2)

1.1.3 Find the equation of the perpendicular bisector of AB. (4)

1.2 The equation of a circle is given as $(x - 1)^2 + (y + 1)^2 = 2(x - y)$.

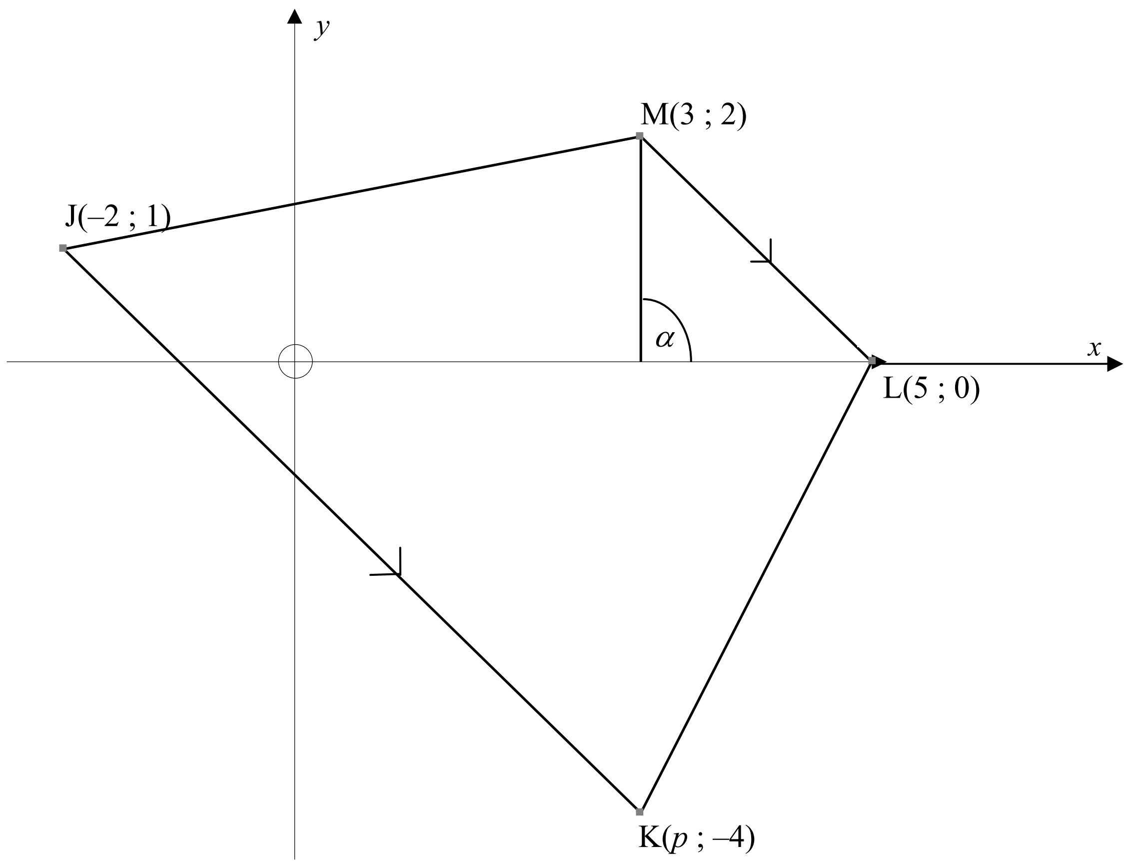
1.2.1 Write down the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)

1.2.2 Hence, write down the co-ordinates of the centre and the length of the radius of the circle. (2)

[15]

QUESTION 2

2.1 In the diagram below, J (–2; 1), K(p; –4), L(5; 0) and M(3; 2) are vertices of trapezium JKLM. Also JK // ML.



2.1.1 Show that $p = 3$. (4)

2.1.2 Calculate JK: LM in the simplest form. (5)

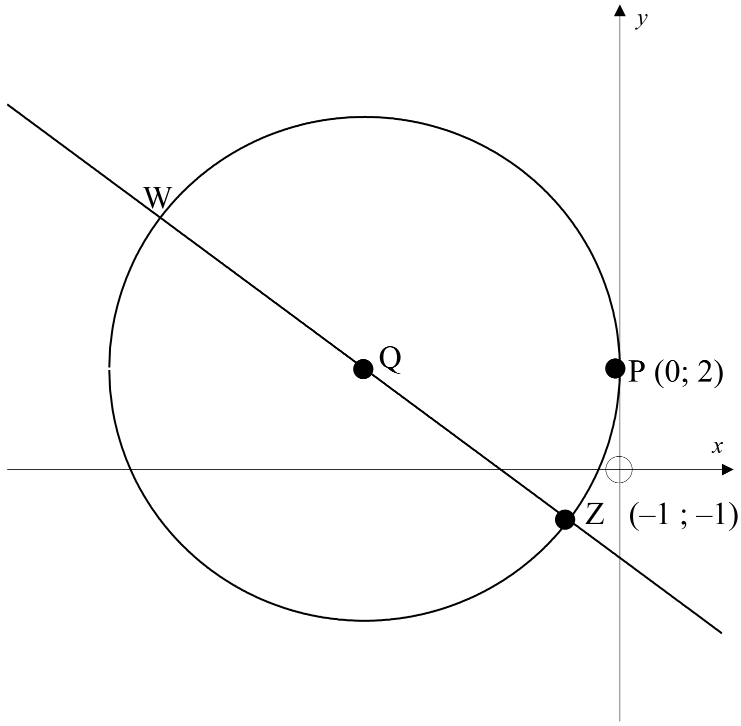
2.1.3 Q (x; y) on JK is such that JQLM is a parallelogram, determine the co-ordinates of Q. (5)

2.1.4 Determine the equation of the line passing through K and M. (2)

2.1.5 Write down the value of α, the inclination of line KM. (1)

2.1.6 If points, R (1; k), J and L are collinear, calculate the value of k. (4)

2.2 In the diagram alongside, centre Q of the circle lies on the straight line $3x + 4y + 7 = 0$. The straight line cuts the circle at W and Z (–1; –1). The circle touches the y - axis at P (0 ; 2).



2.2.1 Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (5)

2.2.2 Determine the length of diameter WZ. (1)
[27]

QUESTION 3

3.1 **Show without using a calculator** that:

$$\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$$ (3)

3.2 Write the following as a single trigonometric ratio:

$$\frac{\tan(180^\circ + \theta).\cos(360^\circ - \theta)}{\sin(180^\circ - \theta).\cos(90^\circ + \theta) + \cos(540^\circ + \theta).\cos(-\theta)}$$ (9)

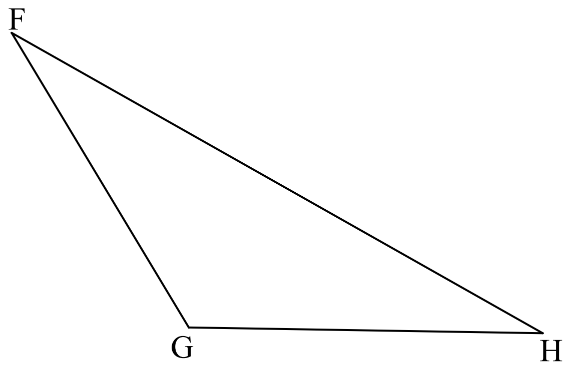
3.3 Prove that: $\sin(45^\circ + x).\sin(45^\circ - x) = \frac{1}{2}\cos 2x$ (5)

3.4 Show $\dfrac{\sin 33^\circ}{\sin 11^\circ} - \dfrac{\cos 33^\circ}{\cos 11^\circ} = 2$ (6)

3.5 Determine the general solution of the equation: $\dfrac{\tan 3x}{\tan 24^\circ} - 1 = 0$ (5)
[28]

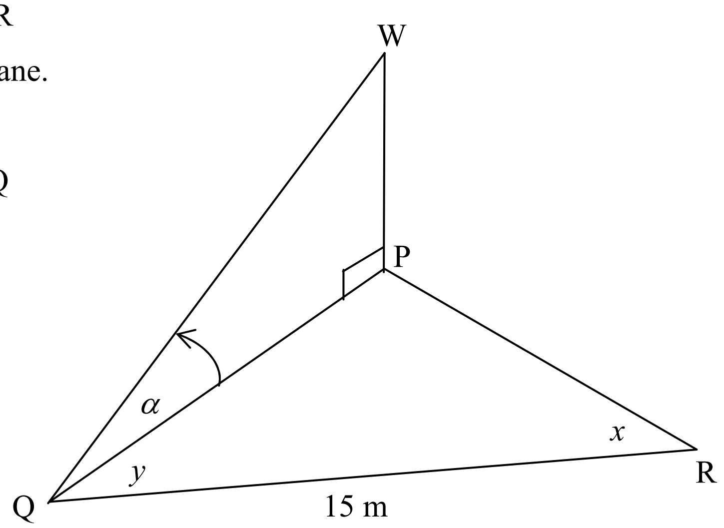
QUESTION 4

4.1 Given Δ FGH with $\hat{G}$ obtuse.



Use the above diagram to prove that: $\dfrac{\sin G}{g} = \dfrac{\sin H}{h}$ (5)

4.2 In the diagram alongside, P, Q and R are points on the same horizontal plane. WP is a vertical telephone mast. The angle of elevation of W from Q is α. $P\hat{Q}R = y$, $P\hat{R}Q = x$ and QR = 15 metres.



4.2.1 Express PW in terms of PQ and α. (2)

4.2.2 Hence, show that $PR = \dfrac{15 \sin y}{\sin (x + y)}$. (4)

4.2.3 If $x = y$, show that $PW = 7,5 \dfrac{\tan \alpha}{\cos y}$ (3)

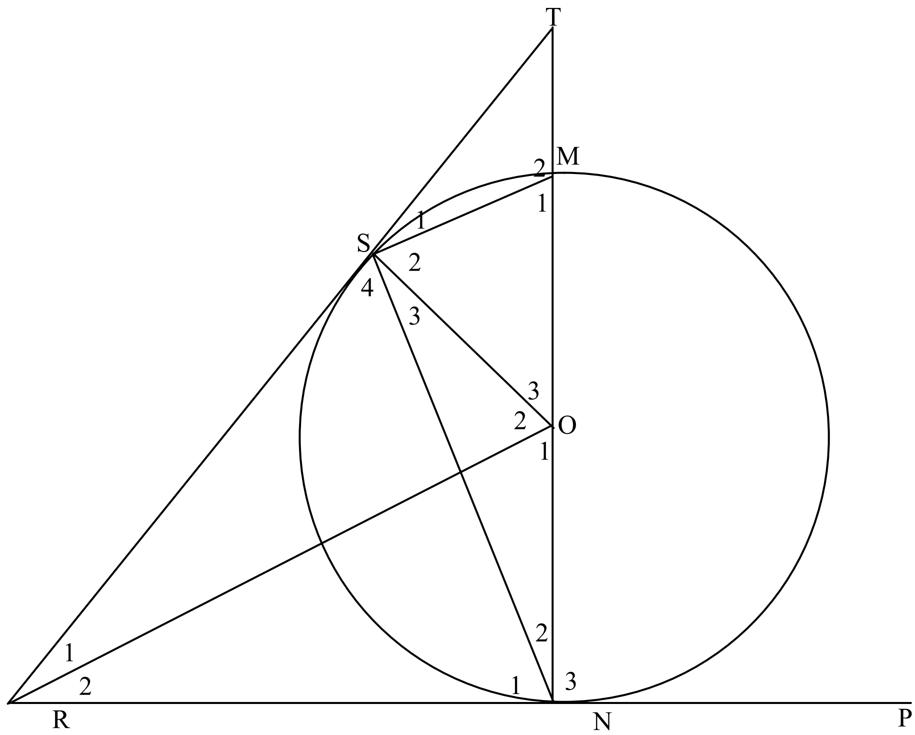
[14]

QUESTION 5

5.1 Complete the following theorem statement:

Opposite angles of a cyclic quadrilateral are … (1)

5.2 In the figure alongside, RS and RNP are tangents to the circle with centre O at the points S and N. Radius NO is produced and cuts the circle at M and meets RS produced at T.



5.2.1 Why is $R\hat{S}O = 90^{o}$? (1)

5.2.2 Prove that RNOS is a cyclic quadrilateral. (4)

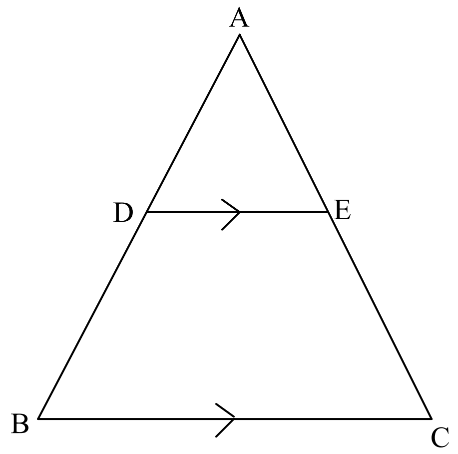
5.2.3 If $\hat{S}_1 = x$, determine, with reasons, FOUR other angles in the figure which are equal to x. (8)

5.2.4 Prove that $\hat{S}_3 = \frac{1}{2}\hat{O}_3$ (4)

[18]

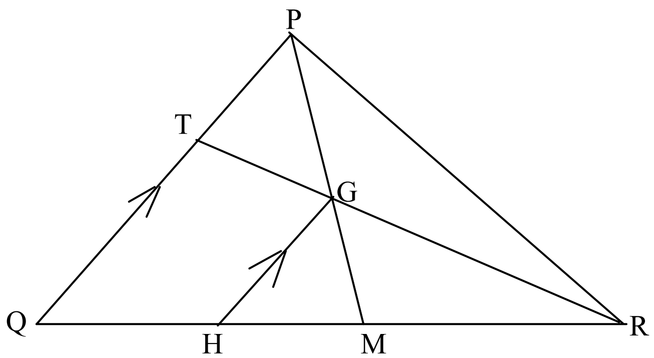
QUESTION 6

6.1 Given Δ ABC with DE // BC as shown in the figure below:



Prove that: $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

6.2 In the diagram below, M is the midpoint of QR in Δ PQR. T is a point on PQ such that PM and TR intersect at G. GH || PQ with H on QR. PG : PM = 1 : 3.

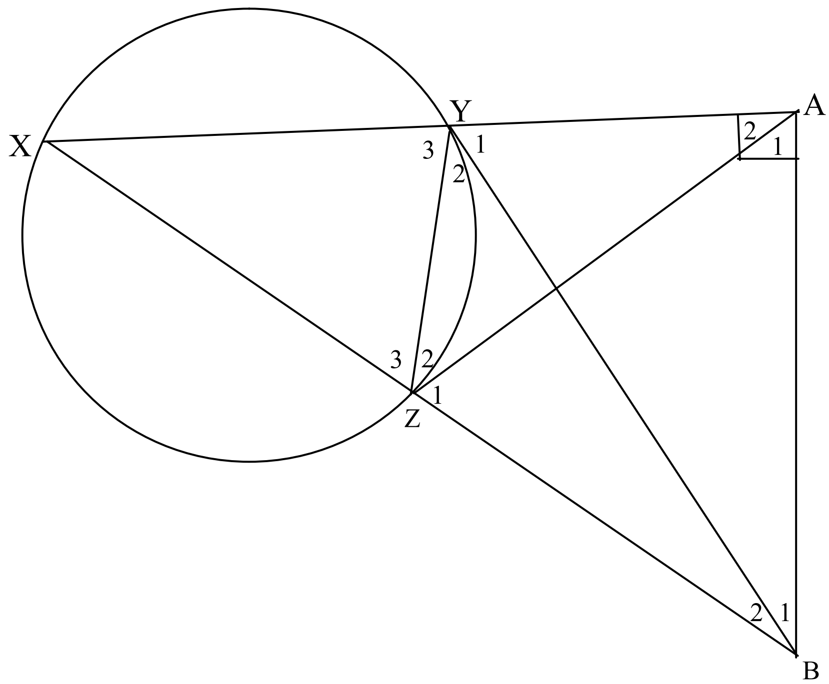


Determine, with reasons, the numerical value of:

6.2.1 $\frac{QH}{HM}$ (3)

6.2.2 $\frac{RG}{RT}$ (5)

6.3 In the diagram below, the chord XY is produced by its own length to A, and tangent AZ is drawn to touch the circle at Z. XZ is produced to meet the line perpendicular to XA at B and AZ = AB.



6.3.1 Prove that $\hat{Z}_3 = 90^{o}$. (5)

6.3.2 Prove that ΔAXZ ||| ΔAZY (3)

6.3.3 Prove that $AZ^2 = AY. XA$. (1)

[23]

TOTAL: 125

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $\quad r \neq 1$ $\quad$ $S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ M$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P\,(A\ or\ B) = P\,(A) + P\,(B) - P\,(A\ and\ B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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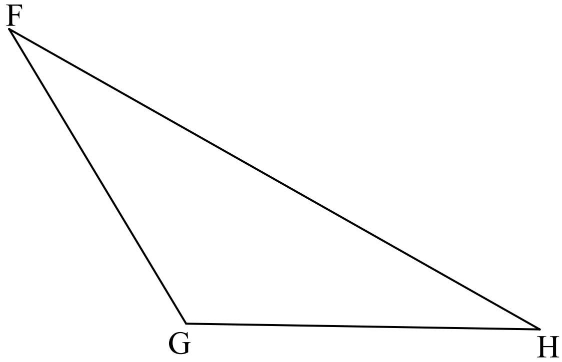
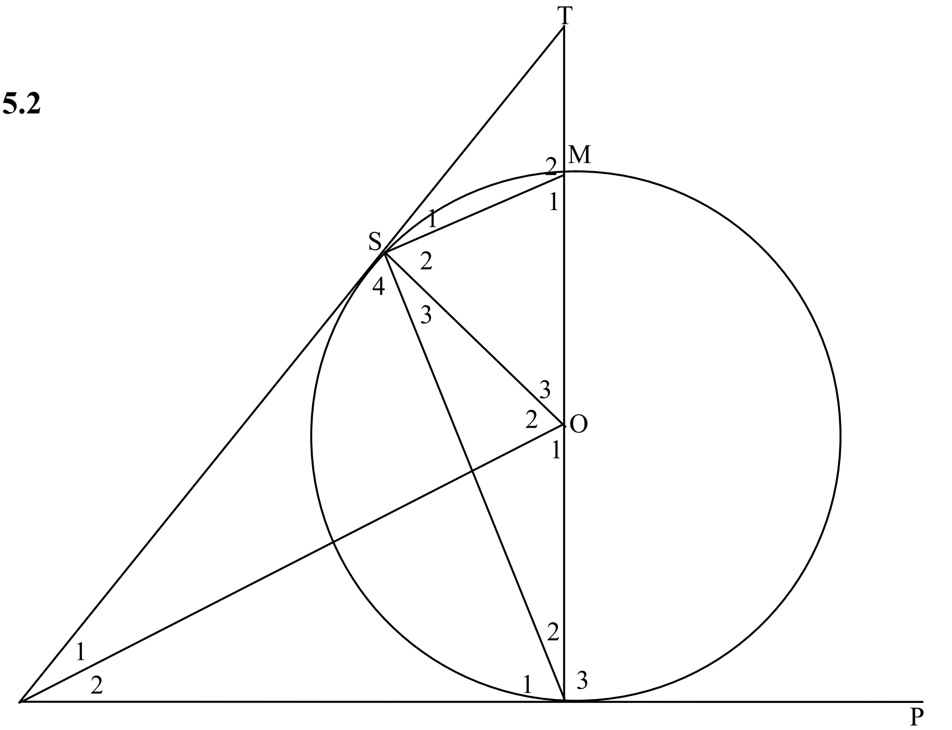
DIAGRAM SHEET 1

QUESTION 4.1



QUESTION 5.2



QUESTION 6.1

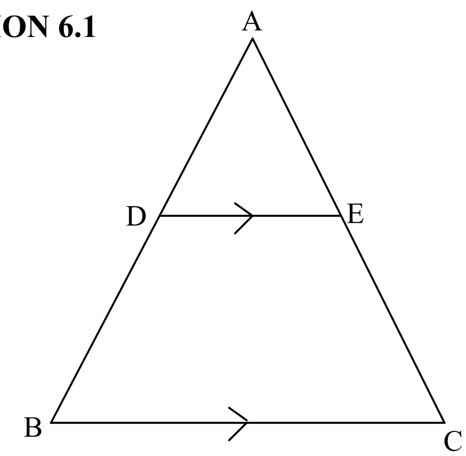


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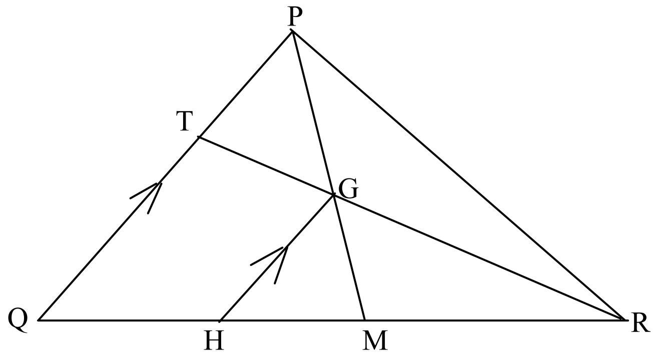
DIAGRAM SHEET 2

QUESTION 6.2



QUESTION 6.3